Quantification of structural robustness : application to the study of a prestressed concrete beam

N.C. KAGHO-GOUADJIO[a], A.D. ORCESI[a], C.F. CREMONA[b], C. MARCOTTE[c]

a. University Paris-Est, IFSTTAR, Materials and Structures Department, Champs-sur-Marne, France

b. SETRA, Technical center for Bridge Engineering, Sourdun, France

c. CETE Nord Picardie, Bridge Division, Haubourdin, France

Abstract:

The term structural robustness gives rise to various definitions and applications. In particular, the European structural standards Eurocodes recommend a structural design to be sufficiently safe against accidental or abnormal loads not explicitly considered in the design. This paper presents a probabilistic approach for the quantification of structural robustness, which measures the impact of a localized failure on the global structural failure. In order to identify the probabilistically most dominant failure mechanisms starting from a local failure, failure tree methods are used, such as the branch and bound method, the β-unzipping method, and an original approach combining the concepts of β unzipping with some bounding process. These methods are used to identify dominant failure paths within reasonable computational times. In particular, it is possible to determine the failure path associated with the largest probability of failure, also called the reference path. Considering this reference path, some robustness indices are proposed to quantify the gap between local and global failures.
The proposed approach is applied to the structural analysis of a prestressed beam. The results obtained with the three methods as well as the computational times required each time are finally compared.

Keywords: **Structural robustness, risk, failure, disproportionate damage**

1 Introduction

In the field of structural engineering, the regulatory framework of Eurocodes defines structural robustness as "the ability of a structure to withstand events like fire, explosions, impact or the consequences of human error, without being damaged to an extent disproportionate to the original cause". This definition highlights the notions of initial damage (local failure) and disproportionate damage (global failure) [1-3]. With such a definition, an essential step is to characterize the transition of the system from a state of local damage (initial failure) to a state of global damage (disproportionate failure). Several examples of structural failure illustrate the impact of local on global damage such as the progressive and partial collapse of the Ronan Point tower in London (UK) in 1968 due to an internal gas explosion at the 18th floor, killing 4 people [4] or the I-35W Bridge collapse in Minneapolis (USA) in 2007 due to a design flaw, killing 13 people and injuring 145 [5] (Figure 1).
When exposed to an accidental action, a structure might loose one or more structural element(s) by failure. The structure is then said to be vulnerable and the degree of vulnerability depends on the extent of the observed direct consequences (damage immediately due to the action of accidental loads). Robustness is estimated taking into account the direct and indirect consequences due to the occurrence of a hazard [6-8]. A simple example is that an impact can cause the failure of a column which is a key element in the structural design (direct consequence). This local failure might then cause the progressive failure of other structural elements until the overall structural failure (indirect effect). Assessing the structural robustness leads to appreciating the gap between the local and the global damage of the system (Figure 1).
With the aim of taking into account multiple uncertainties associated with loads, material and structural properties, this paper proposes a reliability-based approach to quantify structural robustness of civil engineering structures. To analyze structural robustness in a general framework, first, the structural system

should be defined and modeled. Second, the local failure and the global failure should be well characterized to allow the identification of the probabilistically dominant complete failure paths. The local failure may occur at a structural element, a subset of structural elements or a critical area (failure mode). The global failure refers to a critical function of the structural system, which is no longer fulfilled, following the occurrence of a number of localized damage.

To determine the failure path with the largest probability of occurrence starting from the failure of one element, several failure path methods are used. The probabilistically dominant complete failure paths are identified using structural system reliability techniques such as the branch-and-bound method [9], the β-unzipping method [9] and an original hybrid β-unzipping/bounding approach [10-12]. Based on the failure path with the largest probability of occurrence (reference path), several probabilistic robustness indices can be proposed, first by comparing the probability of the system to transit from local to global damage, and secondly by taking into account the notion of risk [7-8], and measuring the gap between the consequences of local and global damage. The study of a prestressed beam is carried out to illustrate the proposed methodology.

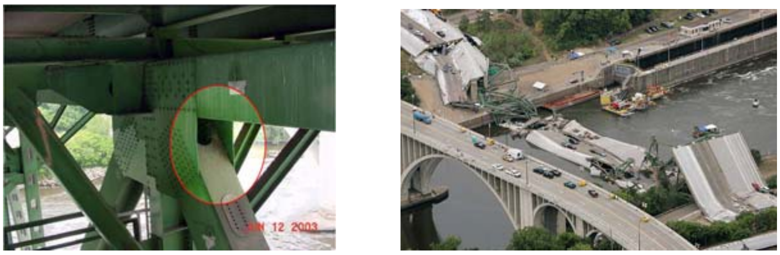

FIG. 1 – Collapse of the Mississipi bridge, Minneapolis, USA, 2007.

2 Measure of structural robustness

Two robustness indicators are introduced in this section, with the aim of determining the extent between local and global damage, as mentioned in the introduction. The first proposed robustness index is a ratio between local and global failure probabilities, as expressed below

$$I_{r,1} = 1 - \frac{P_{global}}{P_{local}} \tag{1}$$

where P_{local} = probability of local failure, and P_{global} = probability of occurrence of the reference failure path The robustness index $I_{r,1}$ varies between the interval $[0;1[$, the lower and upper bounds indicating non robust and robust structures, respectively. The second robustness index is expressed as

$$I_{r,2} = \frac{C_{local} P_{local}}{C_{local} P_{local} + C_{global} P_{global}} \tag{2}$$

This second robustness index $I_{r,2}$ compares the risk (defined herein as the multiplication of the consequence of one scenario by its probability of occurrence) at the local and global scale. This index can also be written as

$$I_{r,2} = \frac{P_{local}}{P_{local} + a P_{global}} \tag{3}$$

where $a = C_{global} / C_{local} \geq 1$ is a ratio between local and global consequences. It is interesting to note that $I_{r,2}$ can be derived from $I_{r,1}$ by the following expression

$$I_{r,2} = \frac{1}{1 + a\left(1 - I_{r,1}\right)} \tag{4}$$

As well as $I_{r,1}$, values of $I_{r,2}$ close to 0 and 1 indicate non robust and robust structures, respectively.

In the following of this paper, some notions and formulations of the reliability approach are used and coupled with the structural analysis. Some of these concepts are briefly reminded in this paragraph. In particular, a finite set of n_v random variables $\mathbf{Z} = (z_1, \ldots, z_n)$ and a performance function M are introduced and defined as $M = g(\mathbf{Z})$ where $g(\mathbf{Z}) < 0$ for $\mathbf{Z}$ in failure set, $g(\mathbf{Z}) > 0$ for $\mathbf{Z}$ in safe set, and $g(\mathbf{Z}) = 0$ for $\mathbf{Z}$ on the limit state surface [13]. The first-order reliability method (FORM) is used in the following to approximate the probability of failure $P_f = P(g(\mathbf{Z}) \leq 0) \approx \Phi(-\beta)$ where β is the reliability index [14] and Φ the standard normal cumulative distribution function.

An interpretation of the robustness indices $I_{r,1}$ and $I_{r,2}$ is proposed thereafter considering a failure path noted q (reference path), which can be for example the probabilistically most dominant failure path. The path q is associated with the sequential order of failure elements that failed $q_1, q_2, \ldots, q_n$, where $q_i =$ failure modes of the path q and $n =$ length of q. It is then possible to characterize local failure with the probability of failure of the first element in the reference path q. Thus $P_{local} = P_f(q_1) = P(g(q_1) < 0)$, with $P_f(q_1) =$ probability of failure of element q_1 and $g(q_1) =$ performance function associated with failure mode q_1. It is also possible to characterize the probability of global failure as $P_{global} = P\left(\bigcap_{i=1}^{n} g^{q_1,\ldots,q_{i-1}}(q_i) \leq 0\right)$ with $g^{q_1,\ldots,q_{i-1}}(q_i) =$ performance function associated with the failure mode q_i knowing that failure modes $q_1, q_2, \ldots, q_{i-1}$ have already occurred, respectively [8]. Obviously, $g^{q_1,\ldots,q_{i-1}}(q_i) = g(q_1)$ for $i = 1$.

It is then possible to write the robustness index $I_{r,1}$ as follows

$$I_{r,1} = 1 - \frac{P\left(\bigcap_{i=1}^{n} g^{q_1,\ldots,q_{i-1}}(q_i) \leq 0\right)}{P(g(q_1) < 0)} \tag{5}$$

which is equivalent to perform the calculation of a conditional probability as below

$$I_{r,1} = 1 - P\left(\bigcap_{i=2}^{n} g^{q_1,\ldots,q_{i-1}}(q_i) \leq 0 \middle| g(q_1) < 0\right) \tag{6}$$

In case where the robustness index $I_{r,2}$ is used, the expression becomes

$$I_{r,2} = \frac{1}{1 + a \times P\left(\bigcap_{i=2}^{n} g^{q_1,\ldots,q_{i-1}}(q_i) \leq 0 \middle| g(q_1) < 0\right)} \tag{7}$$

3 Application to the study of a prestressed concrete beam

In this section, the proposed approach is applied to one of the beams of the Merlebach's VIPP [15]. VIPP are simple span viaducts of precast concrete girders prestressed by post-tension. Figure 2a gives a representation of Merlebach's VIPP, which includes two decks consisting of six isostatic spans of length 32.50 m. Spans of each deck contained five beams each (2.10 m height) spaced 3.15 m and interconnected by a slab of 1.65 m wide and 0.18 m thick. In the beams, the longitudinal prestressing was characterized by 10 STUP cables of cross section characteristics 12Ø8 (Figure 2b), whose the first six were anchored in the butt [15-17].

In this case study, structural robustness is assessed by analyzing crack propagation from a local scale to a global scale of the beam. Explicitly, the local failure is assumed to be the height of a cracked area in the heel of the beam exceeding a critical threshold, and the global failure is assumed to be the total volume of cracked concrete exceeding a pre-defined threshold [12]. The considered beam is simply supported at its two ends, and subjected to a static concentrated load P at mid-span. Modeling of the structure is performed using © SETRA-ST1 software (Figure 3a) which is a calculation software of bar structures [18]. This study presents the modeling of the VIPP beam as developed within the framework of an earlier study on the structure of the Merlebach's VIPP [15] who aimed to take into account the cracking in calculating isostatic prestressed concrete beam and to better understand the deformation of the beam operating in degraded mode.

FIG. 2 – (a) Merlebach viaduct and (b) cross-section of a beam at mid-span.

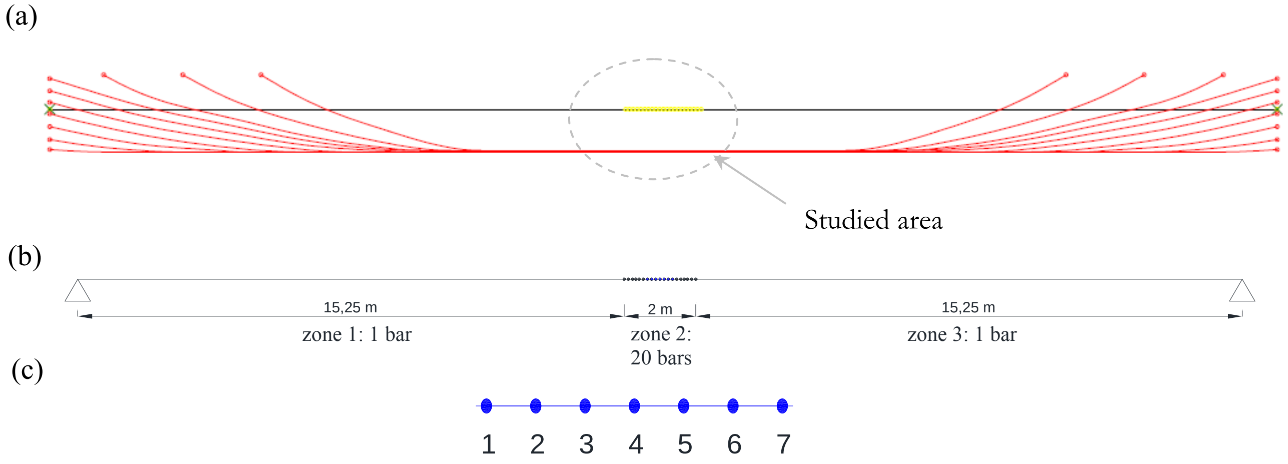

FIG. 3 – (a) Modeling of the beam, (b) studied areas, and (c) detail of the seven sections in zone 2.

The model in Figure 3a represents the prestressed cables in red, the neutral axis of the beam in black, and the studied area (detailed in Figures 3b and 3c) in yellow. Modeling of the beam is affected such that it comprises three areas, the two zones at the ends of a length of 15.25 m, each consisting of one bar, and the central zone of 2 m, consisting of 20 bars (Figure 3b). The area of interest is located at mid-span in the central zone and comprises seven sections numbered 1 through 7 (Figure 3c).

3.1 Modeling of failure

The local failure is assumed to be reached when the crack height $h_{i,crack}$ in the lower fiber of section i is larger than a critical threshold $h_{critical,crack}$. In this case, the geometry of the beam is changed in this section by reducing the heel's height of an inclusive thick in the lower part, which affects the moment of inertia in this particular section. Besides, the global failure considered herein is reached when the volume of cracked concrete v_{crack} calculated in the seven studied sections (see Figure 3) as

$$v_{crack} = e_{section} \times l_{heel} \times \sum_{i=1}^{7} h_{i,crack} \tag{8}$$

is larger than a critical threshold $v_{critical,crack}$, where $e_{section} = 0.10$ m is the spacing between the studied sections and l_{heel} = the width of the heel. In the following, $h_{critical,crack}$ is assumed to follow a lognormal distribution with mean and coefficient of variation (COV) fixed at 0.05 m and 3%, respectively. The mean value of the tension in the prestressing cables σ_{cables} is furthermore taken to be 800 MPa [12]. The statistics of random variables used are provided in Table 1, where σ^{h}_{trac} = limit stress of the concrete in tension outside the study area, σ^{i}_{trac} = limit stress of the concrete in tension in the study area, for the considered section i. Finally, the volume of cracked concrete v_{crack}, which is calculated along the failure path by considering each time the design point values of the parameters in Table 1 when the limit state for critical height is reached (local failure), is compared to the critical volume $v_{critical,crack}$, this latter being a deterministic parameter fixed at 0.04m^3.

Variables	P	ρ	σ_{cables}	σ^{h}_{trac}	σ^{1}_{trac}	σ^{2}_{trac}	σ^{3}_{trac}	σ^{4}_{trac}	σ^{5}_{trac}	σ^{6}_{trac}	σ^{7}_{trac}	$h_{crack,critical}$
Distribution	Normal		Lognormal									
Mean (μ)	0.79 MN	2.5 t/m3	800 Mpa	3.27 Mpa	3.27 Mpa	3.27 Mpa	3.27 Mpa	3.27 Mpa	3.27 Mpa	3.27 Mpa	3.27 Mpa	0.05 m
COV	5%	5%	9%	20%	20%	20%	20%	20%	20%	20%	20%	3%

TAB. 1 – Statistics of random variables.

3.2 Structural robustness analysis

The results obtained by leading the three event-tree approaches are presented in this section and detailed in Table 2 and in Figure 4. The computations of this paper are performed using a work-station with dual quad core Intels Xeons processors and 3.5GB of RAM.

Load P acting at section 4 (mid-span)	$I_{r,1}$	$I_{r,2}$ a=100	Reference path	Probability of failure of the reference path	CPU time (s)
Branch and bound	0.21	0.01	4-7	4.20×10^{-1}	55,500
β-unzipping	0.25	0.01	4-5-3-2-6	3.97×10^{-1}	16,000
β-unzipping with bounding	0.25	0.01	4-5-3-2-6	3.97×10^{-1}	15,900

TAB. 2 – Obtained results.

It appears in this example that the two methods based on β-unzipping are faster to converge compared to the branch and bound method. In this example, there is not a significant gain in calculation times when the bounding process is activated in addition to the β-unzipping even if the number of complete failure paths is significantly reduced (compare Figures 4b and 4c). Also, these results show that the β-unzipping and the β-unzipping with bounding methods identify the same probabilistically most dominant failure path 4-5-3-2-6, with a failure probability $P_f = 3.97\times10^{-1}$. The branch and bound method identifies the reference path 4-7 which is associated with a larger failure probability $P_f = 4.20\times10^{-1}$.

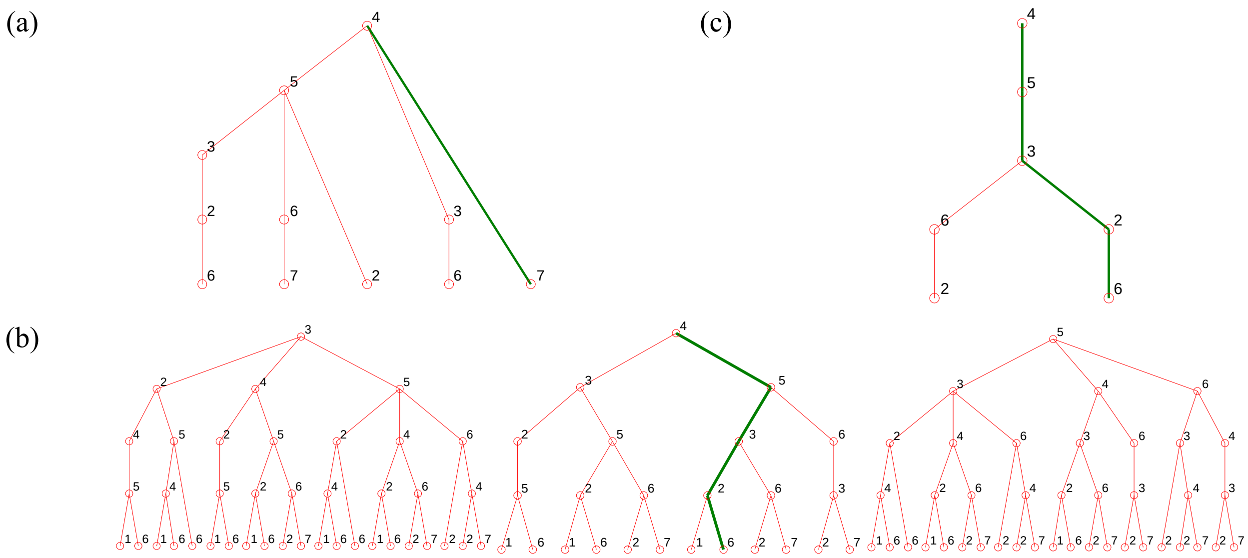

FIG. 4 – Dominant failure paths obtained with (a) branch and bound method, (b) β-unzipping method, and (c) β-unzipping with bounding method.

It is noted that both methods using β-unzipping concepts lead to a symmetric failure path (4-5-3-2-6) with local failures progressively extending from the center of the studied area (where the load is applied) to the extremities of this area. Conversely, the reference failure path obtained with the branch and bound method straightly moves from sections 4 to 7. The difference in those paths can be explained by the fact that whereas the branch and bound considers the probability of failure of the entire path at each node of the tree in Figure 4, the methods using β-unzipping concepts consider at each node the probability of failure of the last element in the failure path. These two paths, even if different, lead to a volume v_{crack} larger than the accepted threshold $v_{critical,crack}$. Finally, the obtained robustness indices $I_{r,1}$ and $I_{r,2}$ are very close with the three different methods. To further illustrate the results, Figures 5a, 5b and 5c show the crack heights in each of the seven sections of the studied area when 4, 4-7, and 4-3-5-2-6 have failed, respectively. In these figures, the values of the variables are those obtained at the design point when the limit state for critical height is reached (local failure) in the last element of the failure path. It is observed that the profile of cracks is more concentrated in the center of the studied area in Figure 5b whereas it is more spread in Figure 5c. These results are in accordance with the difference in the length of the failure path itself. For the failure path 4-7, the global failure is quickly reached and is associated with a sharp and extended crack in section 4. For the failure path 4-3-5-2-6, the failure spread along the different sections which present homogeneous crack heights. Considering the robustness indices, the path 4-7 is obviously associated with the most critical value.

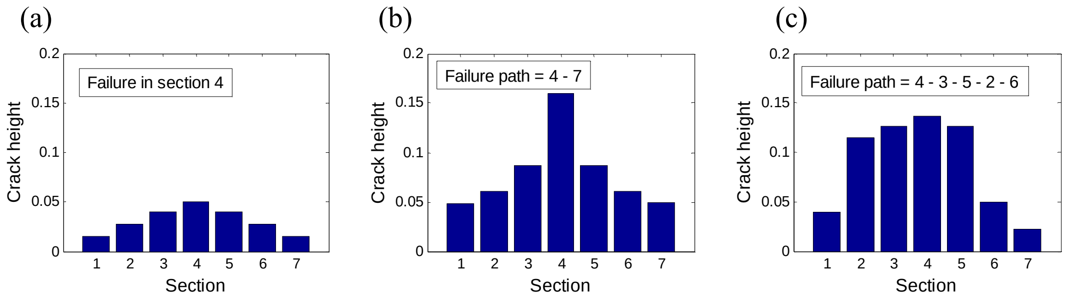

FIG. 5 – Crack heights in each of the seven sections for the failure path (a) 4, (b) 4-7, and (c) 4-3-5-2-6.

4 Conclusions

Design for robustness represents a scientific and technical challenge for civil engineers. A major problem with regard to its incorporation into current methods of design and management is the ability to quantify it. The work presented in this paper proposes a probabilistic approach for the quantification of structural robustness. The proposed approach is based on the study of a series of failure propagation in the structure, to identify the probabilistically most significant global failures and to derive a gap between the probabilities of local and global damage. This approach has been applied in the case of a prestressed concrete beam, which shows the application of the proposed methodology when the concepts of local and global failures can be well characterized and modeled. Further reflection is obviously necessary on the operational implementation of the proposed indices, and their inclusion in a regulatory framework. Also, a thorough sensitivity analysis still needs to be performed to assess how the results of the proposed model are apportioned to the different sources of uncertainty in its inputs.

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